1. Answer any five:

a) If A = (1,2,3) and B = (a,b); List all the members of A×B and B×A.

See Topic: **SET THEORY, Long Answer Type Question No. 1(a).**

b) Solve for x and y from $(\sin x, y) = (0, y^2 - 2)$.

See Topic: **MISCELLANEOUS, Long Answer Type Question No. 1(a).**

c) Give an example of a 1–1 mapping, which is not onto.

See Topic: **SET THEORY, Long Answer Type Question No. 1(b).**

d) Does the set R of all reals from a Group with respect to arithmetic multiplication? Give reasons.

See Topic: **BINARY COMPOSITION, Short Answer Type Question No. 1.**

e) Show that $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\vartheta & \cos\theta \end{pmatrix}^2 = \begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\vartheta & \cos 2\theta \end{pmatrix}$

See Topic: **MATRICES, Short Answer Type Question No. 1.**

f) If $\lim_{x \to a} f(x) = \ell$, show that $\lim_{x \to a} |f(x)| = |\ell|$.

See Topic: **LIMIT, CONTINUITY & DIFFERENTIABILITY, Short Answer Type Question No. 1.**

g) Obtain $\dfrac{dy}{dx}$ when $y = xe^x$.

See Topic: **SUCCESSIVE DIFFERENTIATION, Short Answer Type Question No. 1.**

h) Evaluate $\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3}$.

See Topic: **LIMIT, CONTINUITY & DIFFERENTIABILITY, Short Answer Type Question No. 2.**

i) Find the value of $\int_0^{\frac{\pi}{2}} \dfrac{\sin\theta}{\sin\theta + \cos\theta} d\theta$.

See Topic: **DEFINITE INTEGRALS, Long Answer Type Question No. 1.**

2. a) If R is the set ....find Γ^{-1}.

b) Give the definition of a commutative Group in respect of multiplication.

See Topic: BINARY COMPOSITION, Long Answer Type Question No. 1.

3. a) By Descartes rule of sign show that $x^6 - 3x^2 - x + 1 = 0$ has at least two imaginary roots.

b) If $A, B, ... K$ and $a, b, ... k$ and l are all reals, show that $\dfrac{A^2}{x-a} + \dfrac{B^2}{x-b} + - - + \dfrac{K^2}{x-K} = x + l$ has all

its roots real.

a) **See Topic: POLYNOMIAL, Short Answer Type Question No. 1.**

b) **See Topic: POLYNOMIAL, Long Answer Type Question No. 1.**

4. a) Given that roots of $54x^3 - 39x^2 - 26x + 16 = 0$ are in G.P. Find all the roots.

b) If α, β and γ are the roots of $x^3 + px^2 + qx + r = 0$, find the value of $(i) \sum \alpha^2 \beta$ and $\sum \dfrac{1}{\alpha + \beta}$.

See Topic: POLYNOMIAL, Long Answer Type Question No. 2 & 3.

5. a) If $A - 2B = \begin{pmatrix} 0 & 6 & 2 \\ 6 & -9 & 12 \\ 2 & 9 & -10 \end{pmatrix}$ and $2A + B = \begin{pmatrix} 10 & -3 & 4 \\ 12 & -3 & 4 \\ 4 & 3 & 0 \end{pmatrix}$.

Find matrices A and B.

b) Solve the following system of equations by matrix inversion method. $2x + 5y + 3z = 9$

$$3x + y + 2z = 3$$

$$x + 2y - z = 6$$

See Topic: MATRICES, Long Answer Type Question No. 1 & 2.

6. a) If $f(x)$ = ...$x = 0$.

b) Evaluate (i) $\lim\limits_{x \to 1} \dfrac{(2x-3) \cdot (\sqrt{x} - 1)}{2x^2 + x - 3}$ (ii) $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{x^2}$

See Topic: LIMIT, CONTINUITY & DIFFERENTIABILITY, Long Answer Type Question No. 1.

7. a) If $x = e^t$, and $y = \sin t$ show that $\dfrac{d^2 y}{dx^2}$ at $x = \dfrac{\pi}{2}$ is equal to $-e^{-\pi}$.

b) Expand e^x in ascending powers of x with remainder in Lagranges form.

a) **See Topic: SUCCESSIVE DIFFERENTIATION, Long Answer Type Question No. 1.**

b) **See Topic: FUNCTION OF SEVERAL VARIABLES, Long Answer Type Question No. 1.**

BCA M-134

8. a) Give the definition of $\lim\limits_{(x,y)\to(a,b)} f(x,y)=\ell$ show that $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ does not exist when

$$f(x,y)=\frac{x^2 y}{x^4+y^2} \quad \text{for } x^4+y^2 \neq 0$$

$$=0 \qquad \text{for } x^4+y^2 =0$$

b) If $f(x,y)=xy\dfrac{x^2-y^2}{x^2+y^2}$ if $x^2+y^2 \neq 0$

$$=0 \qquad \text{if } x^2+y^2 =0$$

See Topic: **LIMIT, CONTINUITY & DIFFERENTIABILITY, Long Answer Type Question No. 2 & 3.**

9. a) If $V=\sqrt{x^2+y^2+z^2}$, show that $\dfrac{\partial^2 V}{\partial x^2}+\dfrac{\partial^2 V}{\partial y^2}+\dfrac{\partial^2 V}{\partial z^2}=\dfrac{2}{V}$.

b) If $f(x,y)=x^3+y^3-3x-12y+20$. Obtain maxima and minima points for $f(x,y)$.

a) See Topic: **FUNCTION OF SEVERAL VARIABLES, Long Answer Type Question No. 2.**

b) See Topic: **MISCELLANEOUS, Long Answer Type Question No. 1(b).**

10. a) Evaluate any two of following integrals:

i) $\displaystyle\int \frac{dx}{x^3+1}$ ii) $\displaystyle\int (\log_x)^2\, dx$ iii) $\displaystyle\int \frac{dx}{3+2\sin x}$

b) Show that $\displaystyle\int_0^{\pi/4} \frac{\sin 2x}{\sin^4 x+\cos^4 x}\,dx=\frac{\pi}{4}$.

a) See Topic: **INDEFINITE INTEGRATIONS, Long Answer Type Question No. 1.**

b) See Topic: **DEFINITE INTEGRALS, Long Answer Type Question No. 2.**

11. a) If by a rotation of co-ordinate axes expressions $ax+by$ and $cx+dy$ are transformed into $a'x+b'y$ and $c'x+d'y$, show that $a'd'+b'c'=ad-bc$.

b) The origin is shifted to (3, -1), and then co-ordinate are rotated through angle $\alpha=\tan^{-1} 3/4$; Find co-ordinates of (4, 1) and (5, -2) with respect to new co-ordinate system.

See Topic: **TRANSFORMATION OF CO-ORDINATES, Long Answer Type Question No. 1 & 2.**

12. Reduce the following equation to the canonical form and determine the nature of the conic represented by it. $8x^2-12xy+17y^2+16x-12y+3=0$

See Topic: **GENERAL EQUATION OF SECOND DEGREE, Long Answer Type Question No. 1.**

BCA M-135